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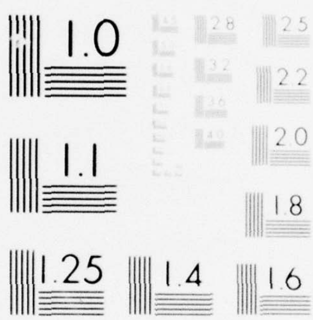

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CAUSES OF VARIANCE IN MAXIMUM ENTROPY SPECTRAL ANALYSIS.

M. D. Green

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Maximum entropy methods are currently in use for the estimation of spectral quantities. Although several authors have mentioned excessive variance in this type of estimation, no definitive study on the matter has yet been published. This report investigates the variance in the frequency estimate as a function of signal-to-noise ratio, sample offset, initial phase, and the order of the autoregressive process. A discussion of power estimation is also included. Comparisons with Fourier analysis is included where appropriate.

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SUMMARY

Problem

In recent years there has been increasing interest shown in data-adaptive signal processing techniques. Of special interest has been a method now known as maximum entropy methods, or MEM for short.

While MEM has been shown to be superior to standard Fourier techniques in separation of very close frequencies, no definitive studies have been published which analyzed the variance in the spectral estimates obtained. This report investigates this aspect of MEM analysis.

Results

Maximum entropy methods of spectral analysis have been analyzed as to the effects of autoregressive (AR) process order (M), signal to noise (S/N), dc offset, frequency, and initial phase. The results show that the location of the spectral peak (LP) is a function of each of these save dc offset. In particular, and in contrast to Fourier analysis, the LP bias is a function of signal-to-noise ratio. It is also heavily dependent upon the order of the AR process. For the particular case of $M = 16$, $S/N = 1000$, Fourier techniques bias is less in magnitude and leads MEM bias by $\pi/2$ radians. For both MEM and Fourier transform, the effect of initial phase is to shift the bias phase by a corresponding amount. Power estimation requires integration limits which are functions of S/N, location of adjacent spectral peaks, and the AR order, and it is thus a difficult parameter to estimate.

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CONTENTS

INTRODUCTION

Maximum entropy methods of spectral analysis (MEM) are currently in use at NOSC and at other facilities, and some concern has been noted about excessive variance in the estimation of spectral quantities using this method. Several authors have mentioned such variance (Ref. 1–3), and this report presents an experimental investigation into the problem.

Since the MEM algorithms are by now well established in the literature, this report does not include the theoretical development of the relationship between entropy and autoregressive (AR) processes. References 2 and 4 provide a very good theoretical discussion, and Reference 5 gives a very usable, programmable algorithm as well as an excellent discussion of the development of MEM.

EXPERIMENTAL PARAMETERS

The time domain data studies consisted of a cosine wave at a fixed frequency and phase mixed with normally distributed random noise. The frequencies studied were arbitrarily chosen to range between 2/N and 3/N hertz, inclusive. Phases were 0.0, $\pi/4$, and $\pi/2$ radians. The record length N was 64 samples, with a Nyquist frequency of 0.5 Hz. The sample mean was always subtracted from the data in order to allow a better evaluation of the effects of dc offset. Signal-to-noise power ratios (S/N) were 0.5, 2.0, 50.0, 200.0, and 5000.0. No attempt was made to predict a correct AR process order (see Ref. 6 and 7), but the effect of order was studied.

EXPERIMENTAL RESULTS

The basic information being sought in this study was an estimation of the location of spectral peak (LP) and the variance of the estimates about a mean value. Of the parameters expected to influence LP, namely S/N, frequency, sample offset, initial phase, and AR process order, the first and the last must be presented together.

Order of the Autoregressive Process

The initial concern was the effect of the AR process order (M) on the spectrum. It has been reported both that the correct order is crucial (Ref. 2), and that little impact is noticed within certain ranges (Ref. 1). Spectral computations were made for each of two frequencies using three different S/N's of 2.0, 50.0, and 5000. The values of M and the calculated biases are tabulated against S/N in Table 1. Bias is defined to be the deviation from the true LP, as computed from an infinite, continuous wave. From the table, two factors are immediately apparent. First, for a "fundamental" frequency (a multiple of 1/N), bias effects are much more consistent than for an "off" frequency. In fact, for the second frequency shown, the worst S/N of 2.0 has the estimator bias changing sign over an absolute range of 30.5×10^{-4} Hz, while the first frequency is at least uniformly of the same sign. Even at an S/N of 5000, the frequency has a range of 6.31×10^{-4} Hz, while the fundamental frequency has a range of 0.88×10^{-4} Hz. The second feature is the variation for a given M at either frequency as S/N is reduced. In some cases, as for M = 20 for frequency = 2/N Hz, the bias actually increases with increasing S/N, but the more general condition shows a decrease in bias with increasing S/N. It should be noted here that each tabulated value of bias represents only a single noise realization, rather than an ensemble average, and that, for a given M, the noise realization is the same. Because of monetary constraints, a detailed statistical evaluation of the effect of AR order was not possible. However, to confirm the apparent variation caused by the relationship between M and S/N, ten realizations were averaged for each of four frequencies, for selected S/N ratios. An AR order of 16 was chosen. The results are shown in Fig. 1. It should be noted that only statistically significant estimates are shown on the graph. Table 1 and Fig. 1 show that for MEM analysis, bias is a function both of signal-to-noise ratio and AR order. This is not the case for Fourier spectral analysis.

Bias Estimation for High Signal-to-Noise Ratio

As a comparison with Fourier techniques (FT), the bias generated by proximity to the zero frequency axis was investigated. For Fourier techniques, the LP is contaminated by the side lobes of the negative frequency component. The contamination, or bias, takes the form of a damped sinusoid, decreasing in magnitude as one leaves zero frequency. Since it has been shown that bias, in the MEM case, is not only a function of signal to noise, but is also inconsistent, a high signal-to-noise ratio of 5000 was used for this study. The results are shown in Fig. 2 for M = 16. As can be seen, for these parameters the MEM and FT biases are approximately $\pi/2$ radians apart, and the MEM bias is considerably greater in magnitude. Further, although not enough frequency points were used, it appears that the bias is itself biased toward the positive.

Initial Phase Effects

Figure 2 also shows the effect of three different initial phases for both MEM and FT for two frequencies. As can be seen, for both methods, a change in initial phase results in an equal shift in bias phase.

Effects of Sample Offset

As noted earlier, the sample mean was removed from the data in all cases to facilitate an understanding of the effects of nonzero mean in the time domain data. A constant "dc" offset of 1.0 was added to time samples at frequencies 2/N Hz and 2.37/N Hz. No appreciable effects were discernible in the LP for the MEM analysis for either frequency.

Power Estimation

In the course of this investigation an attempt was made to estimate the power in the various frequency components in the waveform. This necessitates an integration process since the AR method is basically an energy density approach. Unfortunately, no valid results were obtained. In order to estimate the energy (or power), appropriate limits of integration are required. While for very high signal-to-noise ratios it is possible to empirically determine relatively acceptable limits, these limits are functions of signal to noise, proximity to other peaks in the spectrum, and the order of the AR process.

DISCUSSION

MEM analysis has been shown by several authors to be a valuable tool in estimating the frequency components of a complex waveform. By its very nature of being data-adaptive it tends to "find" the spectral quantities rather more rapidly than Fourier techniques. However, it has several drawbacks which might tend to limit its usefulness: there is no exact way to determine precisely the correct AR order; the estimates are biased by high signal to noise; it is quite difficult to determine appropriate frequency limits of integration when attempting to estimate power. None of these problems occur with Fourier spectral analysis.

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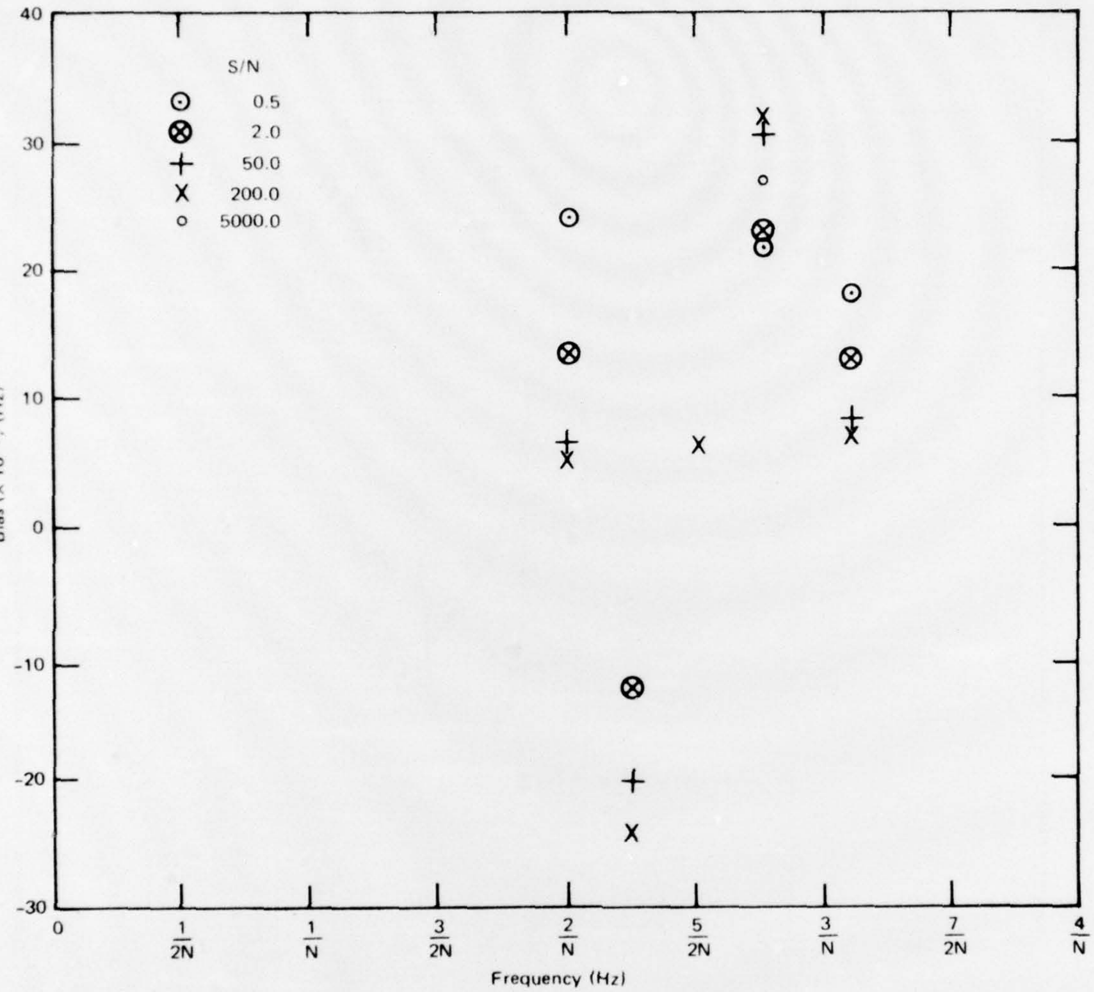

Figure 1. MEM estimates of bias for various S/N ratios.

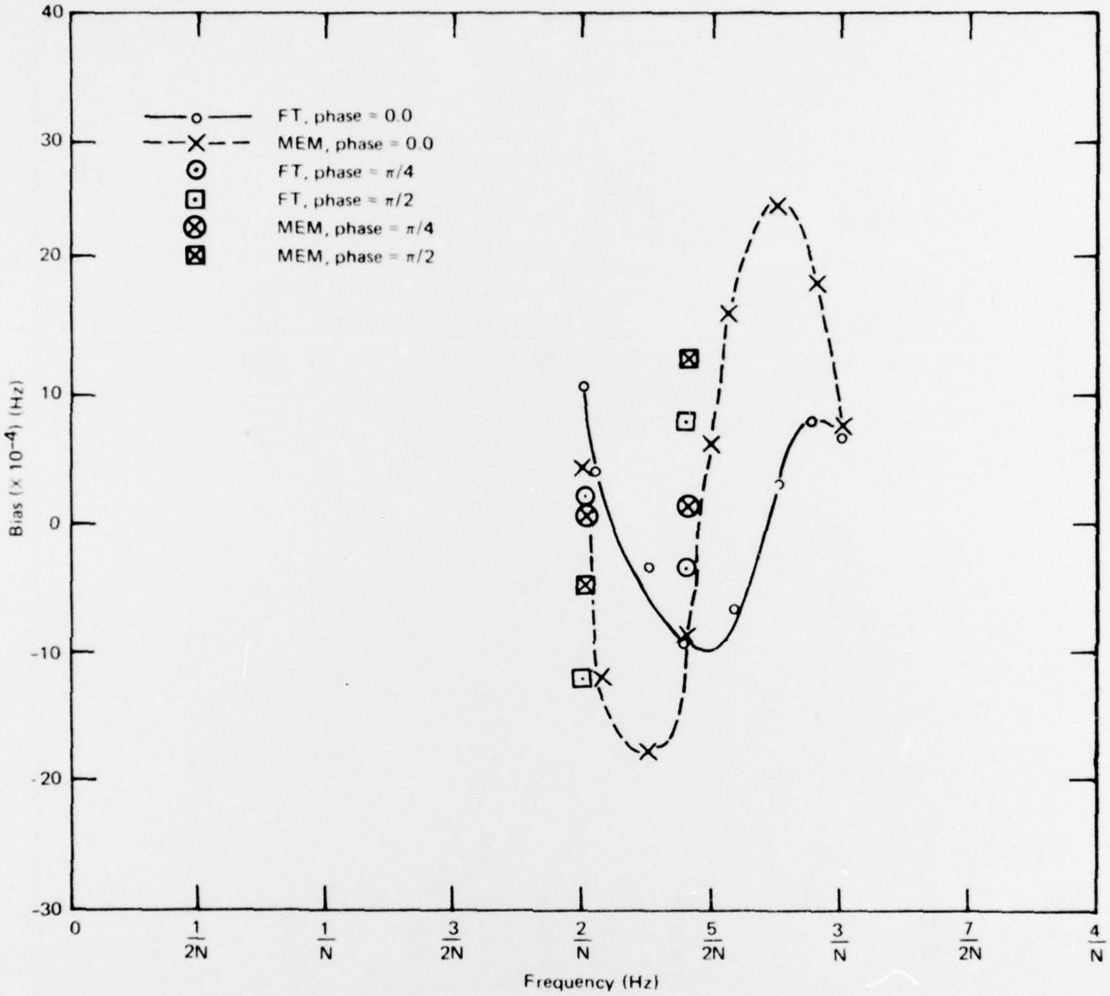

Figure 2. MEM and Fourier technique estimates of bias; S/N = 5000.

Table 1. Effect of AR order (M) on bias estimation.

M	S/N	Bias ($\times 10^{-4}$)(Hz)	S/N	Bias ($\times 10^{-4}$)(Hz)	S/N	Bias ($\times 10^{-4}$)(Hz)
Nominal Frequency = 2/N = 0.03125 Hz						
8	5000	4.98	50	7.36	2.0	30.02
10	5000	4.83	50	7.13	2.0	36.55
12	5000	4.91	50	6.73	2.0	37.15
14	5000	5.02	50	6.19	2.0	19.98
18	5000	4.82	50	5.02	2.0	16.82
20	5000	4.78	50	5.02	2.0	2.49
22	5000	4.38	50	7.05	2.0	13.41
24	5000	4.42	50	6.81	2.0	6.41
26	5000	4.29	50	6.23	2.0	4.76
28	5000	5.17	50	5.34	2.0	16.25
Nominal Frequency = 2.37/N = 0.03703125 Hz						
8	5000	-9.21	50	-22.17	2.0	-5.50
10	5000	-7.59	50	-11.16	2.0	8.66
12	5000	-7.66	50	-7.41	2.0	11.27
14	5000	-8.89	50	-15.63	2.0	-3.34
18	5000	-7.69	50	-5.67	2.0	7.05
20	5000	-8.03	50	-6.84	2.0	-7.61
22	5000	-9.03	50	-13.83	2.0	-19.23
24	5000	-10.25	50	-13.06	2.0	-8.69
26	5000	-11.35	50	-13.41	2.0	-17.98
28	5000	-5.04	50	-3.40	2.0	6.36

Appendix

TABULATION OF RESULTS

The following table presents a comparison between FT and MEM for various S/N ratios for the frequencies indicated in Fig. 1 and 2. "FT σ_M" and "MEM σ_M are, respectively, the standard deviations of the Fourier bias and the MEM bias. The heading "no. of σ's" represents the number of standard deviations of the bias. A value under this heading which is greater than about 3 may be considered statistically significant. The AR order is 16, and the number of realizations, N, is 10.

Table A-1. Statistical comparison of FT and MEM bias.

S/N	FT Bias ($\times 10^{-4}$) (Hz)	FT σ_M ($\times 10^{-4}$) (Hz)	No. of σ's	MEM Bias ($\times 10^{-4}$) (Hz)	MEM σ_M ($\times 10^{-4}$)(Hz)	No. of σ's
Frequency = 0.03125 Hz; phase = 0.0 radians*						
0.5	-0.883	21.772	0.04	24.051	4.4721	5.31
2.0	7.281	2.498	2.92	13.540	1.325	10.22
50.0	10.523	0.455	23.13	5.984	0.191	31.33
200.0	10.836	0.228	47.53	5.234	0.115	45.51
5000.0	11.031	0.044	250.71	4.904	0.024	204.33
Frequency = 0.03220 Hz; phase = 0.0 radians*						
0.5	-5.664	5.875	0.96	12.33	5.216	2.36
2.0	1.594	2.084	0.765	-1.686	2.069	0.82
50.0	3.953	0.396	9.98	-11.454	0.482	23.76
200.0	4.172	0.197	21.18	-11.469	0.281	40.82
5000.0	4.336	0.387	11.20	-11.146	0.207	53.85
Frequency = 0.03516 Hz; phase = 0.0 radians*						
0.5	-9.313	4.339	2.15	1.568	6.507	0.24
2.0	-4.820	1.947	2.48	-12.932	3.352	3.66
50.0	-3.195	0.374	8.54	-19.889	2.095	9.49
200.0	-3.039	0.190	16.0	-24.007	2.311	10.39
5000.0	-2.914	0.004	728.5	-17.644	2.484	7.10
Frequency = 0.03711 Hz; phase = 0.0 radians*						
0.5	-7.867	6.548	1.20	-2.865	7.669	0.37
2.0	-9.281	2.120	4.38	-13.804	4.470	3.10
50.0	-8.577	0.383	22.4	-9.564	1.440	6.64
200.0	-8.492	0.189	44.93	-9.085	0.803	11.31
5000.0	-8.445	0.038	222.20	-8.203	0.210	39.06
Frequency = 0.03906 Hz; phase = 0.0 radians*						
0.5	-7.258	6.758	1.07	1.405	8.036	0.18
2.0	-8.899	2.657	3.35	-3.251	4.820	0.68
50.0	-9.641	1.393	6.92	5.652	5.017	1.13
200.0	-9.664	0.218	44.33	6.171	0.897	6.88
5000.0	-9.711	0.004	2427.75	6.741	0.223	30.23

* DC offset of 1.0 added to bias.*

(Contd)

Table A-1. (Continued)

S/N	FT Bias (× 10^{-4}) (Hz)	FT σ_M (× 10^{-4}) (Hz)	No. of σ's	MEM Bias (× 10^{-4}) (Hz)	MEM σ_M (× 10^{-4}) (Hz)	No. of σ's
Frequency = 0.04297 Hz; phase = 0.0 radians*						
0.5	11.505	5.083	2.26	22.041	5.529	3.99
2.0	7.255	2.629	2.76	23.020	3.191	7.21
50.0	4.029	0.539	7.48	30.571	1.142	26.77
200.0	3.662	0.269	13.61	32.693	0.806	40.56
5000.0	3.380	0.053	63.77	25.724	0.469	54.85
Frequency = 0.04688 Hz; phase = 0.0 radians*						
0.5	15.711	4.835	3.25	18.230	4.942	3.69
2.0	10.844	2.412	4.50	13.531	1.850	7.31
50.0	8.102	0.498	16.27	7.909	0.307	25.76
200.0	7.805	0.245	31.86	7.279	0.157	46.36
5000.0	7.563	0.054	140.06	6.592	0.066	99.88
Frequency = 0.03125 Hz; phase = $\pi/4$ radians*						
5000.0	2.242	0.063	35.59	1.032	0.048	21.5
Frequency = 0.03125 Hz; phase = $\pi/2$ radians*						
5000.0	-11.953	0.053	225.53	-4.647	0.082	56.67
Frequency = 0.03711 Hz; phase = $\pi/4$ radians*						
5000.0	-2.976	0.037	80.43	1.783	0.173	10.31
Frequency 0.03711 Hz; phase = $\pi/2$ radians*						
5000.0	8.220	0.060	137.	12.925	0.285	45.35
Frequency = 0.03125 Hz; phase = 0.0 radians*						
5000.0	11.031	0.044	250.71	4.904	0.025	196.16
Frequency = 0.03711 Hz; phase = 0.0 radians*						
5000.0	-8.445	0.037	22.59	-8.203	0.210	39.06

* DC offset of 1.0 added to bias.*